2023

COMMERCE

Paper : COMHG4046

(Business Mathematics)

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct alternatives from any *six* of the following : 1×6=6

(a) If $\begin{vmatrix} 5x & 10 \\ 2 & 4 \end{vmatrix} = 0$, then the value of x is

(i) –1

(ii) 0

(iii) 1

(iv) 2

(b) If $A = [a_{ij}]$ is a skew-symmetric matrix, then A should be a

(i) square matrix

(ii) rectangular matrix

(iii) row matrix

(iv) column matrix

(c) $\frac{d}{dx}(\log x)$ is equal to

(i) $\log x$

(ii) x

(iii) $-x$

(iv) $\frac{1}{x}$

(d) $\int (MC)\,dx$ is equal to

(i) MC

(ii) AC

(iii) TC

(iv) MR

(e) The present value V of perpetual annuity A is given by

(i) $V = \frac{A}{i}$

(ii) $V = A(1+i)^n$

(iii) $V = \frac{A}{i}[1+(1+i)^{-n}]$

(iv) $V = \frac{A}{i}[1-(1+i)^{-n}]$

where, i = Rate of interest per unit sum per annum, n = Number of years.

(f) In linear programming problem, which of the following is true?

(i) Both objective function and constraints are linear.

(ii) Objective function is linear and constraints are non-linear.

(iii) Objective function is non-linear and constraints are linear.

(iv) Both objective function and constraints are non-linear.

(g) In linear programming problem, optimum value of objective function, if exist, is obtained at

(i) any point of the feasible region

(ii) any point on the boundary of the feasible region

(iii) any point inside the boundary of the feasible region

(iv) one of the corner points on the boundary of the feasible region

(h) The cofactor of the element 4 in the determinant

$$\begin{vmatrix} 3 & -14 & 1 \\ 5 & 4 & -10 \\ -2 & 10 & -1 \end{vmatrix}$$

is

(i) 96

(ii) −1

(iii) 58

(iv) −25

(i) If A and B be two matrices so that the product AB is defined, then $(AB)^T$ is equal to

(i) AB

(ii) $-AB$

(iii) $A^T B^T$

(iv) $B^T A^T$

(j) $\frac{d}{dx}(a^x)$ is equal to

(i) a^x

(ii) $a^x \log_e a$

(iii) $a^x \log_x a$

(iv) $a^x \log_a x$

2. Answer any *five* of the following questions : 2×5=10

(a) Define a diagonal matrix with an example.

(b) Find the interest on ₹ 3,000 @8% per annum simple interest for the period from 15th August, 2010 to 27th October, 2010.

(c) For what value of x, $A = \begin{bmatrix} 5 & x \\ -2 & 4 \end{bmatrix}$ will be singular?

(d) If

$$f(x) = \log\frac{1-x}{1+x}$$

show that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$.

(e) If $y = (x+2)(x+1)^2$, find $\frac{dy}{dx}$.

(f) Evaluate the integral, $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$.

(g) Find x and y if $\begin{bmatrix} 7 & x+y \\ x-y & -6 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 8 & -6 \end{bmatrix}$.

3. Answer any *six* of the following questions : 5×6=30

(a) Prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

(b) If

$$A - 2B = \begin{bmatrix} -7 & 7 \\ 4 & -8 \end{bmatrix}$$

and $A - 3B = \begin{bmatrix} -11 & 9 \\ 4 & -13 \end{bmatrix}$. Find the matrices A and B.

(c) (i) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} 1-x; & \text{when } 0 \le x \le 1 \\ x-1; & \text{when } x > 1 \end{cases}$$

Find $f\left(\frac{1}{2}\right)$. Is $f(x)$ continuous at $x = 1$? 3

(ii) Evaluate

$$\lim_{x\to 0} \frac{x}{\sqrt{2+x} - \sqrt{2-x}}$$ 2

(d) If $y = x^3 \log\frac{1}{x}$, prove that

$$x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$$

(e) If $u = 2(ax+by)^2 - (x^2+y^2)$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4(a^2+b^2-1)$$

(f) Discuss the graphical method of solving linear programming problem.

(g) A certain sum compounded annually amount to ₹ 2,420 in 2 years and to ₹ 2,662 in 3 years. Find the principal and rate of interest.

(h) Evaluate the following integral :

$$\int x\sqrt{2x+1}\,dx$$

(i) From the first principle, find the derivative of $\sqrt{x^2+a^2}$ with respect to x. If

$$A=\begin{bmatrix}1 & 0 & -2\\ 2 & 2 & 4\\ 0 & 0 & 2\end{bmatrix}$$

verify $A^2-3A+2I=0$.

(j) A sinking fund is created for the redemption of debentures of ₹ 1,00,000 at the end of 25 years. How much money should be provided out of profits each year for the sinking fund, if the investment can earn interest @4% p.a.? [Given, $\log 1{\cdot}04=0{\cdot}0170$, $\text{antilog}\,0{\cdot}425=2{\cdot}661$]

4. Answer any *two* of the following questions : 10×2=20

(a) (i) Solve the following system of simultaneous linear equations by Cramer's rule : 6

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$3x+2y+9z=14$$

(ii) A person buys 20 kg of rice, 10 kg of atta, 5 kg of pulses, 4 kg of sugar and 2 kg of salt for a month. Rice costs ₹ 16 per kg, atta costs ₹ 15 per kg, pulses costs ₹ 35 per kg, sugar costs ₹ 21 per kg and salt costs ₹ 8 per kg. Using matrix multiplication, find the amount of money spent by the person. 4

(b) The total cost C of output x is given by

$$C=300x-10x^2+\frac{1}{3}x^3$$

Find the output level at which the marginal cost and average cost attain their respective minima. 10

(c) (i) Verify Euler's theorem for the function $u=\dfrac{xy}{x+y}$. 5

(ii) The marginal revenue function of producing x units of a product is given by $(8+5x-x^2)$. Find the Total Revenue (TR) function and the Average Revenue (AR) function. Given TR = 82 when $x = 5$. 5

(d) (i) If the cost of a machine is reduced from ₹ 10,000 to ₹ 1,250 in 3 years, find the rate of depreciation. Given, $\log 1250 = 3 \cdot 0969$, $\log 5 = 0 \cdot 6990$. 5

(ii) A man borrows ₹ 10,000 at 5% compound interest and agrees to pay both the principal and the interest in 10 equal installments at the end of each year. Find the amount of these installments. Given $(1 \cdot 05)^{-10} = 0 \cdot 6139$. 5

5. Answer any *one* of the following questions : 14

(a) (i) What is linear programming problem? Define objective function, feasible region and feasible solution associated with linear programming. 2+1+2+2=7

(ii) Solve the following LPP by graphical methods : 7

Minimize $z = 3x + 2y$

subject to

$$5x + y \geq 10$$
$$x + y \geq 6$$
$$x + 4y \geq 12$$
$$x \geq 0,\ y \geq 0$$

(b) (i) Find the maximum and minimum values of $2x^3 - 21x^2 + 36x - 20$. 7

(ii) Evaluate $\int x^2 e^x dx$. 7

(c) A manufacturer produces two types of products X and Y. Each product is first processed in a machine M_1 and then sent to another machine M_2 for finishing. Each unit of X requires 20 minutes time on M_1 and 10 minutes on M_2 while the corresponding times for Y are 10 minutes on M_1 and 20 minutes on M_2. The total time available on each machine is 6000 minutes. Calculate the number of units of X and Y produced by constructing a matrix equation of the form $AX = B$ and then solve by matrix inversion method. 14

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